

# A Practical Approach to Quantum Annealing

GOTO CHICAGO 2020

**TRIFORK**  
...think software



AGENDA

# Practical Quantum Annealing - Part 2

Constraint Satisfaction Problems Revisited

---

Networks and Network Algorithms

---

Divide and Conquer - An Introduction

---



# Objective function for optimisation - bias

Source: <https://www.dwavesys.com/quantum-computing>



# Objective function for optimisation - bias

$$\mathbf{E}_{ising}(s) = \sum_{i=1}^N h_i s_i + \sum_{i=1}^N \sum_{j=i+1}^N J_{i,j} s_i s_j$$

OR

$$\mathbf{E}_{qubo}(a_i, b_{i,j}; q_i) = \sum_i a_i q_i + \sum_{i < j} b_{i,j} q_i q_j .$$

BINARY QUADRATIC MODEL



# Binary Quadratic Model (BQM)

```
import dimod

bqm = dimod.BinaryQuadraticModel(
    {0: 1, 1: -1, 2: .5},          # Linear
    {(0, 1): .5, (1, 2): 1.5},    # Quadratic
    1.4,                           # Offset
    dimod.Vartype.SPIN)           # SPIN/BINARY
```



## BINARY CSP

# dwavebinarycsp

CONSTRAINT 1

$$a = b$$

CONSTRAINT 2

$$b \neq c$$

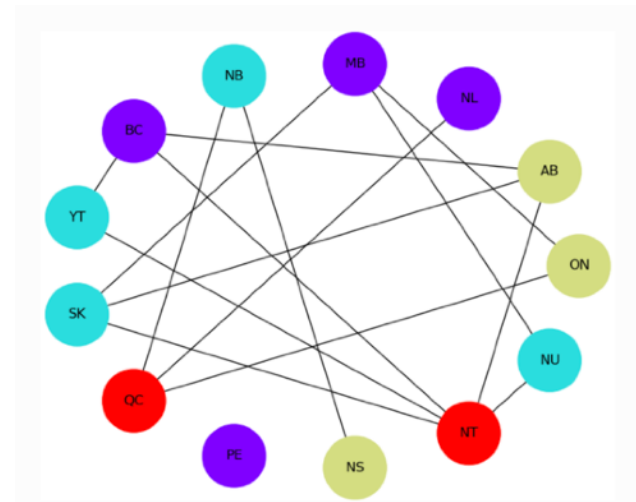
```
import dwavebinarycsp
import operator

csp=dwavebinarycsp.ConstraintSatisfactionProblem('BINARY')
csp.add_constraint(operator.eq, ['a', 'b'])
csp.add_constraint(operator.ne, ['b', 'c'])
result = csp.check({'a': 1, 'b': 1, 'c': 0}) # True
```



# Formulating map colouring as a CSP

- View the map as a graph
  - Each area is a node
  - Each border is a vertex
- Area color as a binary constraint
  - **(qc\_red, qc\_yellow, qc\_green, qc\_blue)** can only be **(0,0,0,1), (0,0,1,0), (0,1,0,0), (1,0,0,0)**
- Two neighbouring area cannot have same color
  - **!(qc\_red && on\_red)**
  - **!(qc\_green && on\_green)**
  - ..
  - **!(qc\_red && nb\_red)**
  - ..



© 2016 D-Wave Systems Inc. All Rights Reserved



# *Demo*

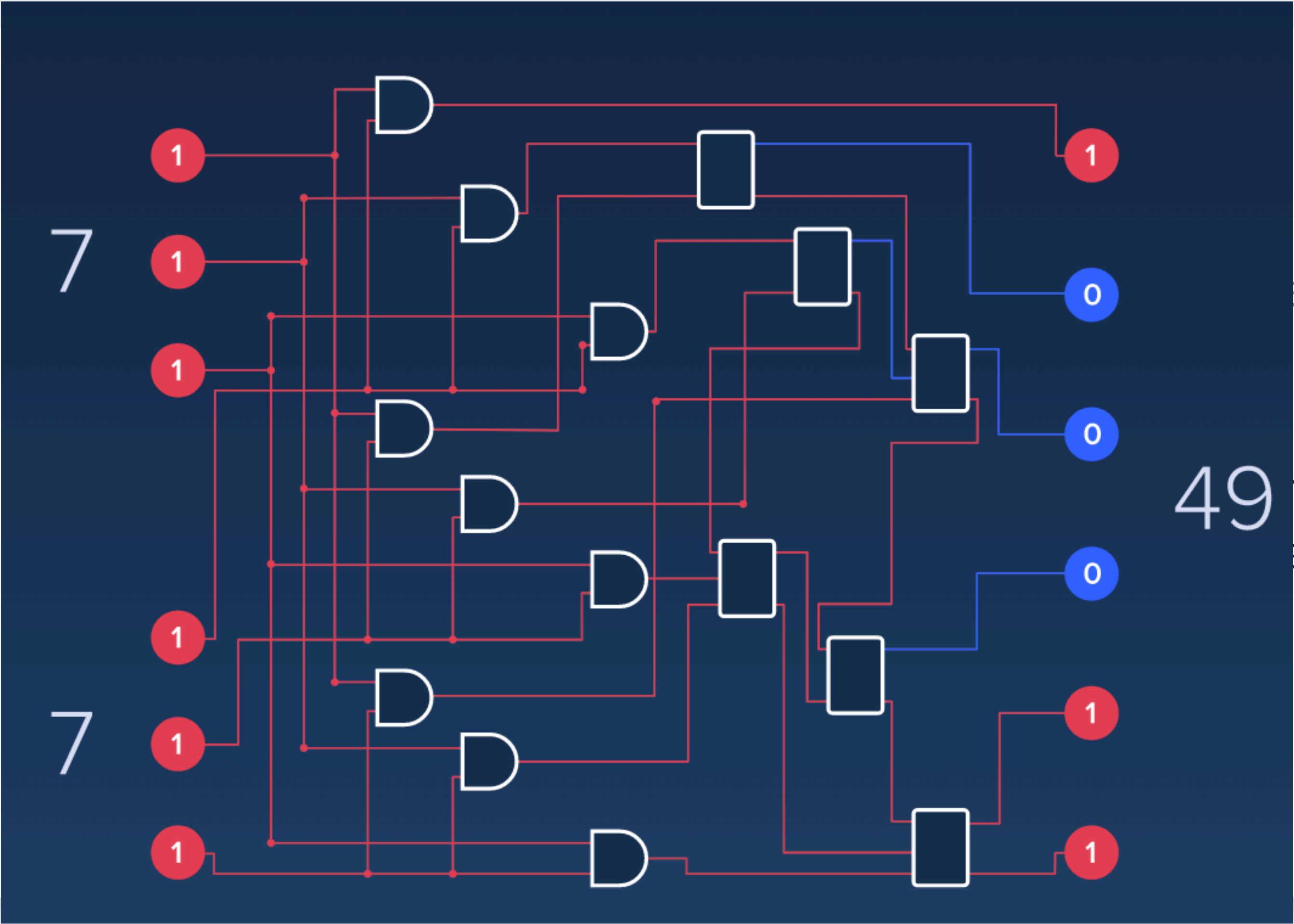
MAP COLORING





# Example: Factoring

```
def factor(P)
  csp = dwav
  bqm = dwav
  # we know
  p_vars = [
  # convert
  fixed_vari
  fixed_vari
  # fix prod
  for var, v
    bqm.fi
```



e variables

```
mat(P))
ables.items() }
```



AGENDA

# Practical Quantum Annealing - Part 2

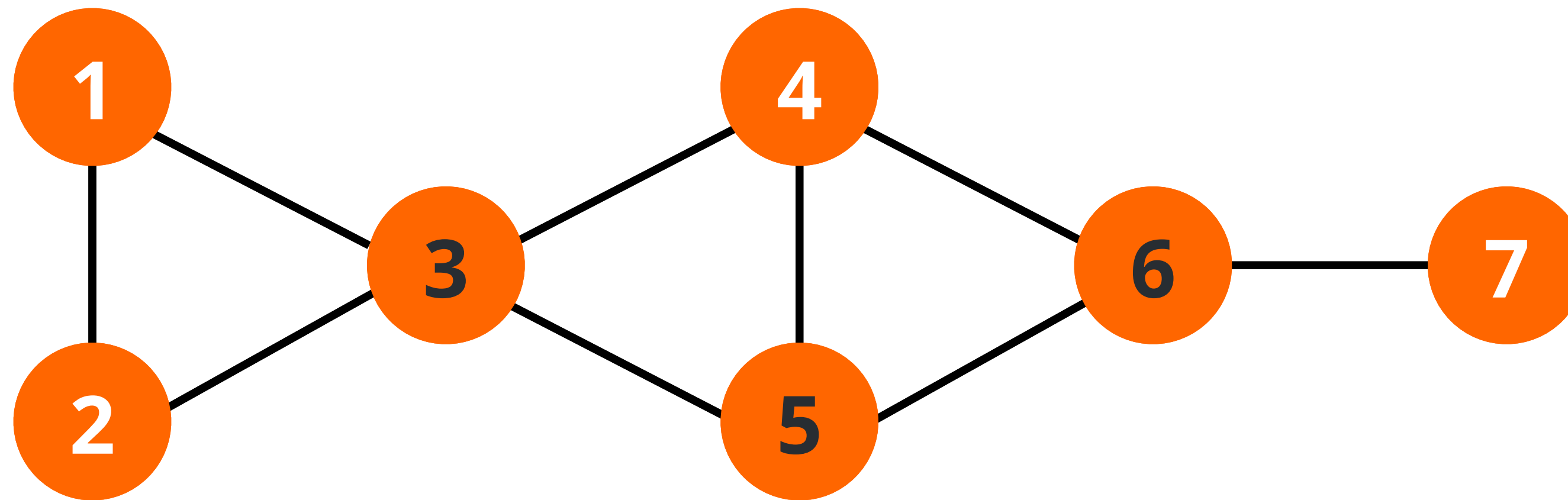
Constraint Satisfaction Problems Revisited

Networks and Network Algorithms

Divide and Conquer - An Introduction



# Example: Vertex cover



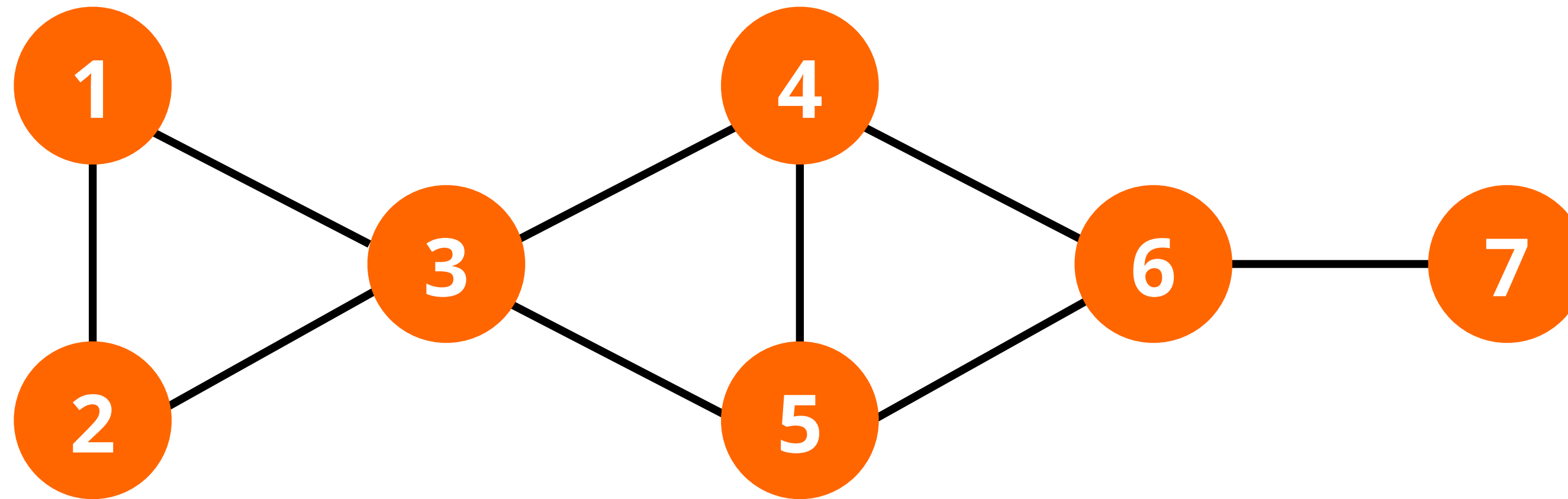
# Objective function for optimisation

- Construct a function with bias that favour an optimal solution
- Nodes
  - Favor nodes with more connectors
- Connectors
  - As few as possible





# Objective function for optimisation



LINEAR

$$-x_1 - x_2 - 3x_3 - 2x_4 - 2x_5 - 2x_6 + 0x_7 +$$

QUADRATIC



$$x_1x_2 + x_1x_3 + x_2x_3 + x_3x_4 + x_3x_5 + \\ x_4x_5 + x_4x_6 + x_5x_6 + x_6x_7$$



# Objective function for optimisation

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

LINEAR

$$-x_1 - x_2 - 3x_3 - 2x_4 - 2x_5 - 2x_6 + 0x_7 +$$

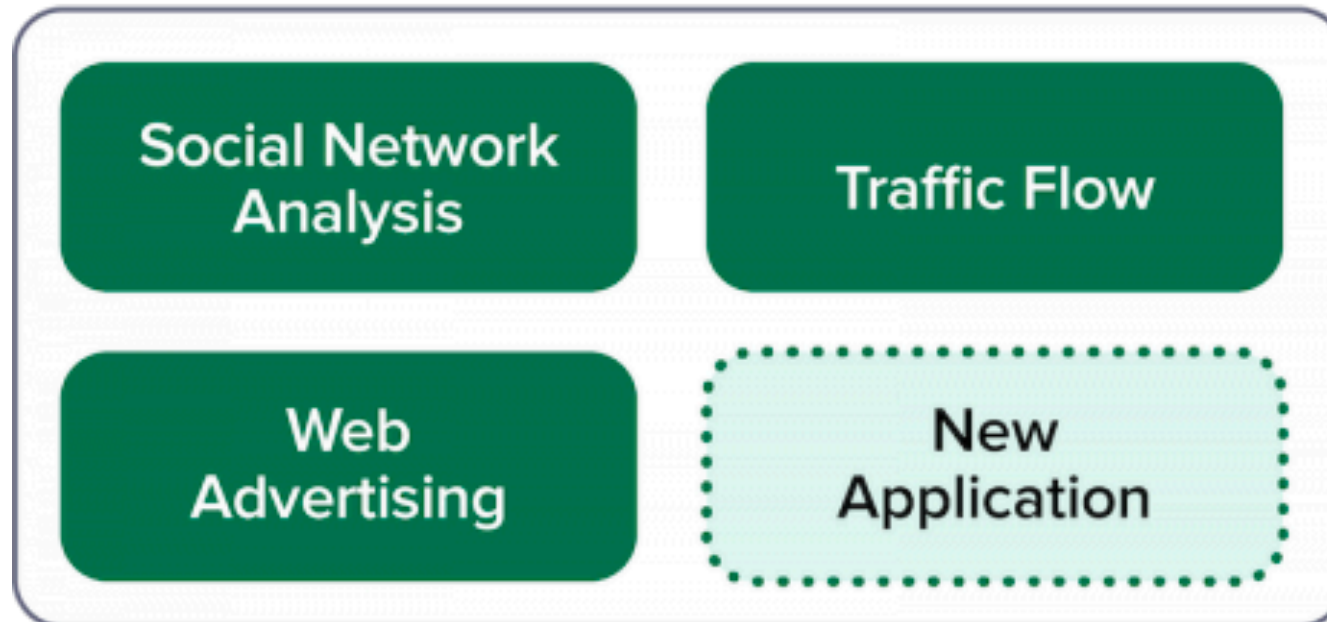
QUADRATIC



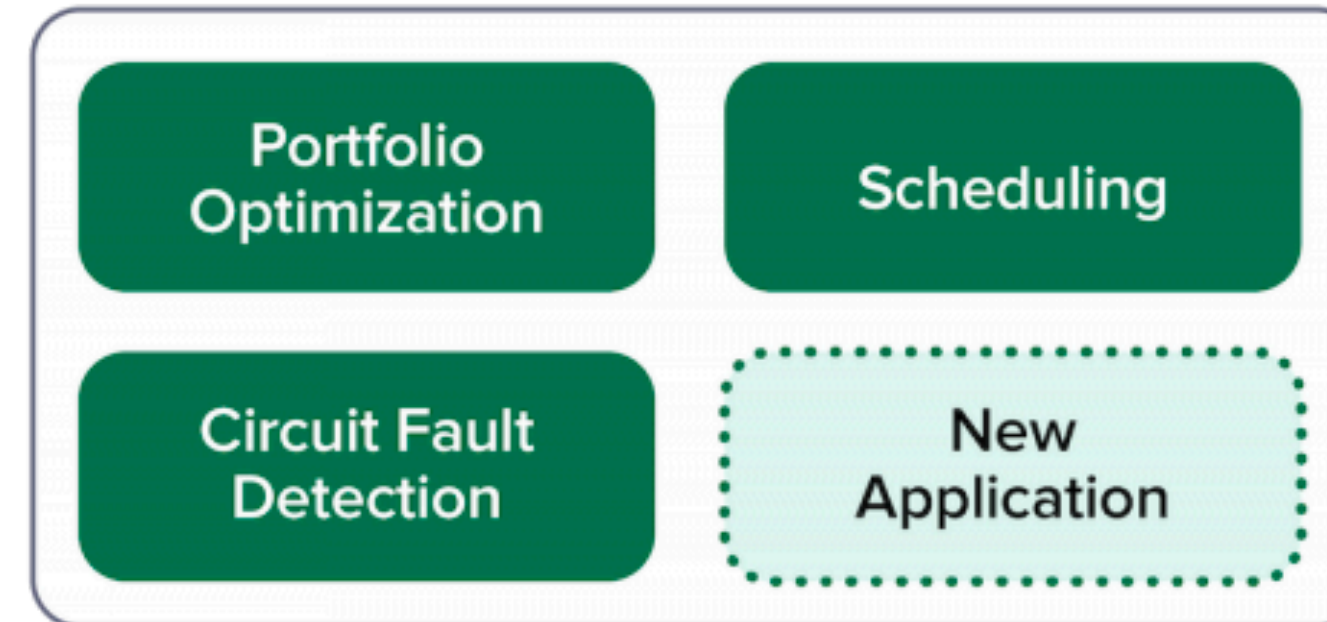
$$x_1x_2 + x_1x_3 + x_2x_3 + x_3x_4 + x_3x_5 + \\ x_4x_5 + x_4x_6 + x_5x_6 + x_6x_7$$



## Optimization

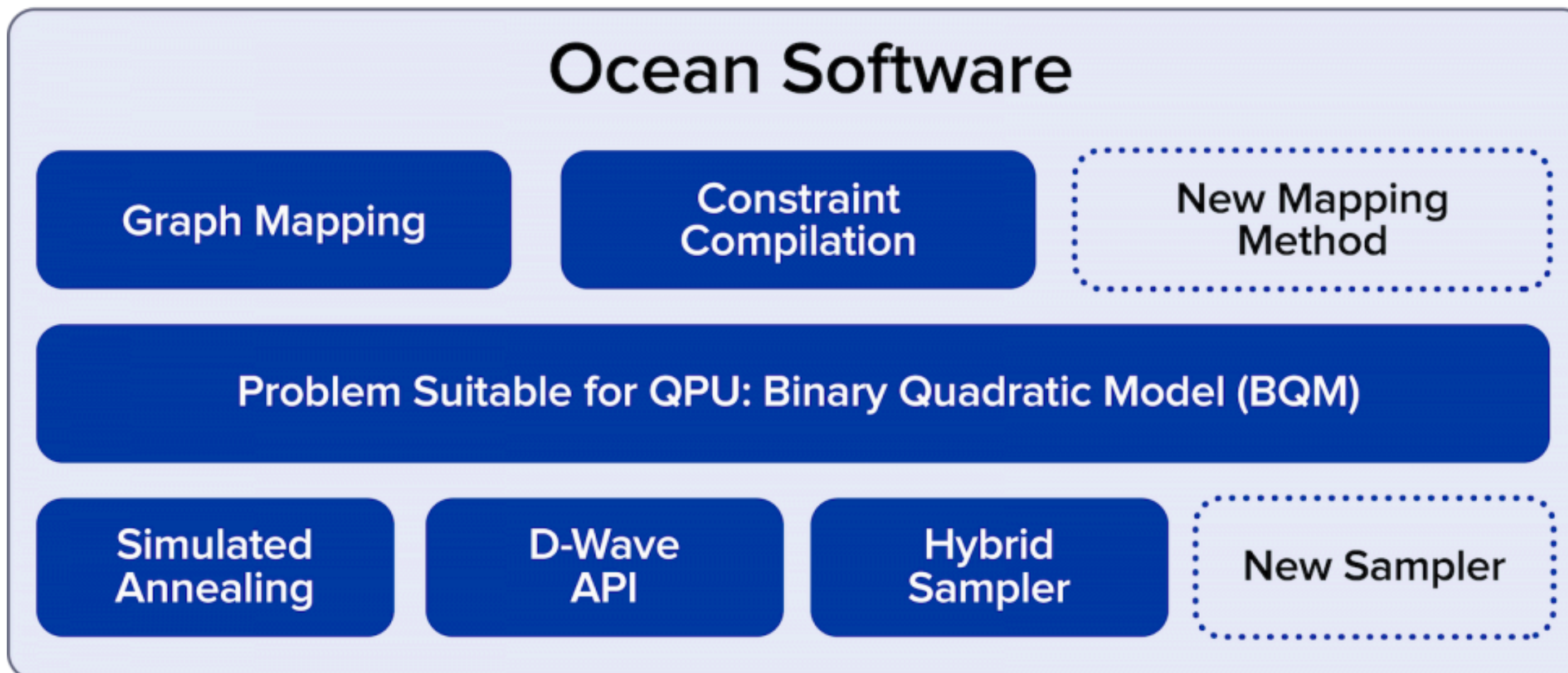


## Constraint Satisfaction



Applications

## Ocean Software



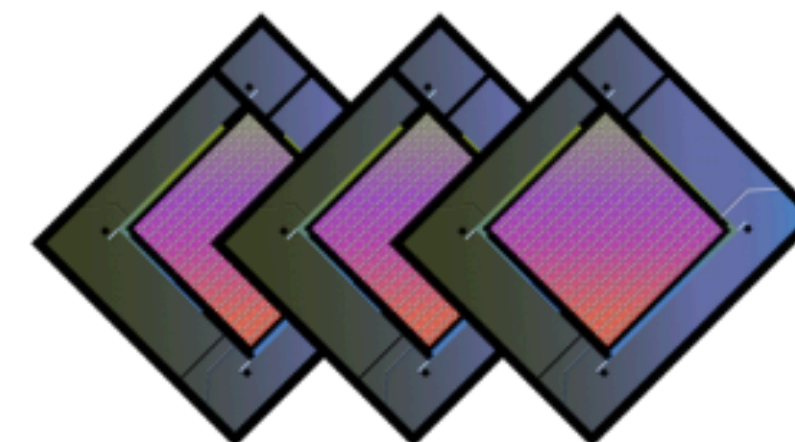
Mapping  
Methods

Uniform  
Sampler API

Samplers



CPU and GPU



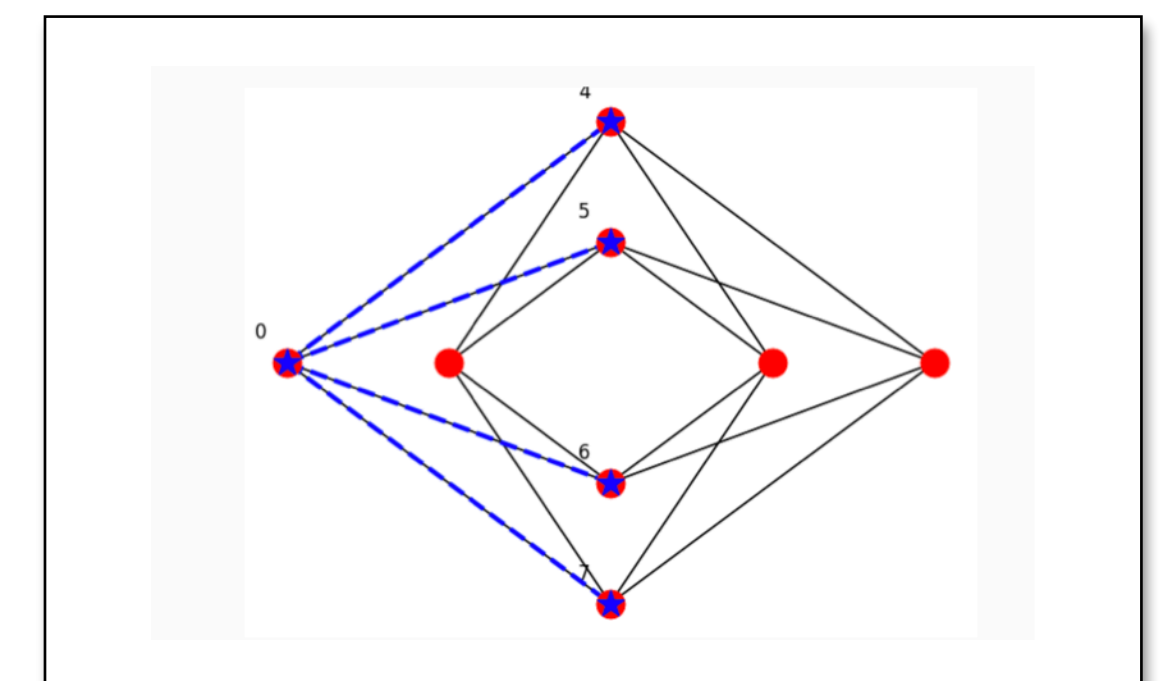
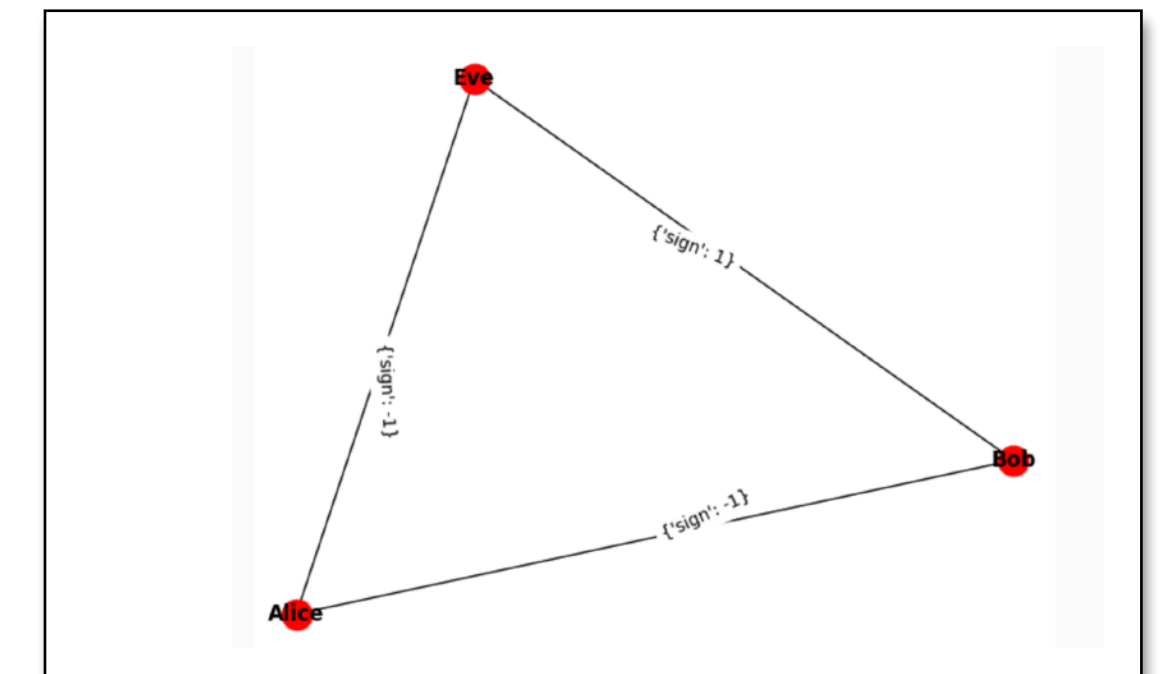
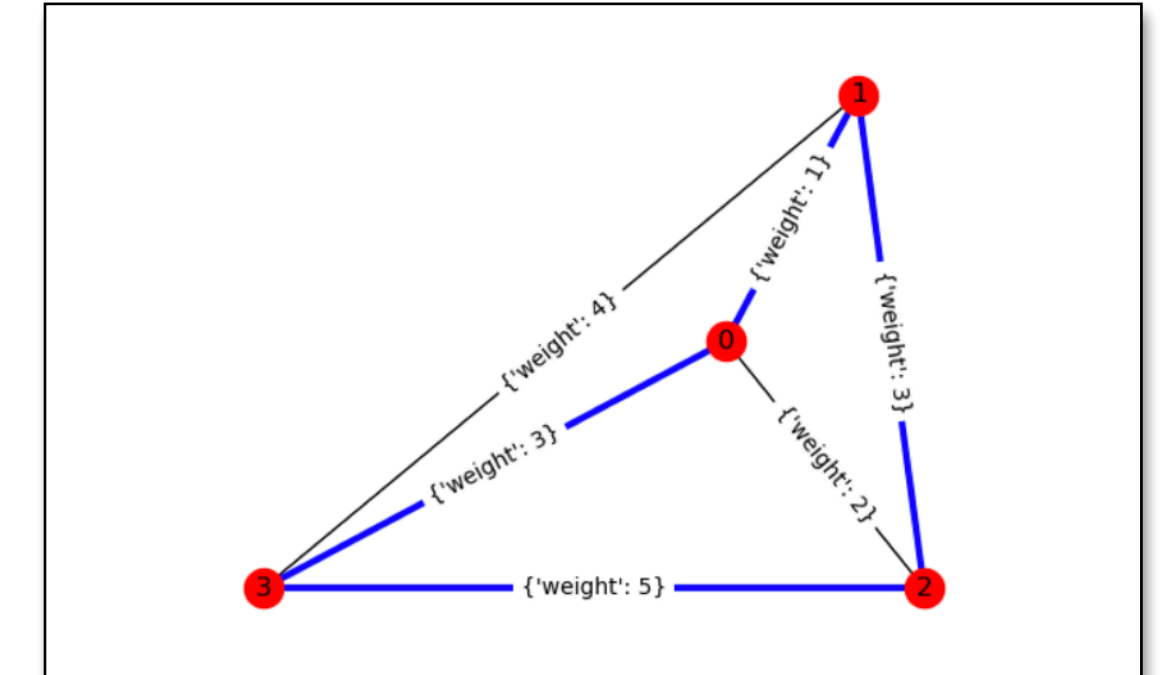
QPUs

Compute  
Resources



# dwave-networkx

- Algorithms
  - Canonicalization
  - Clique
  - Coloring
  - Cover
  - Elimination Ordering
  - Markov Networks
  - Matching
  - Maximum Cut
  - Independent Set
  - Social
  - Traveling Salesperson
- Drawing
  - Chimera Graph Functions Graph Generators
  - D-Wave Systems
  - Other Graphs
- Utilities
  - Decorators
  - Graph Indexing
  - Exceptions
- Default sampler
  - Sampler API
  - Functions





AGENDA

# Practical Quantum Annealing - Part 2

Constraint Satisfaction Problems Revisited

---

Networks and Network Algorithms

---

Divide and Conquer - An Introduction

---

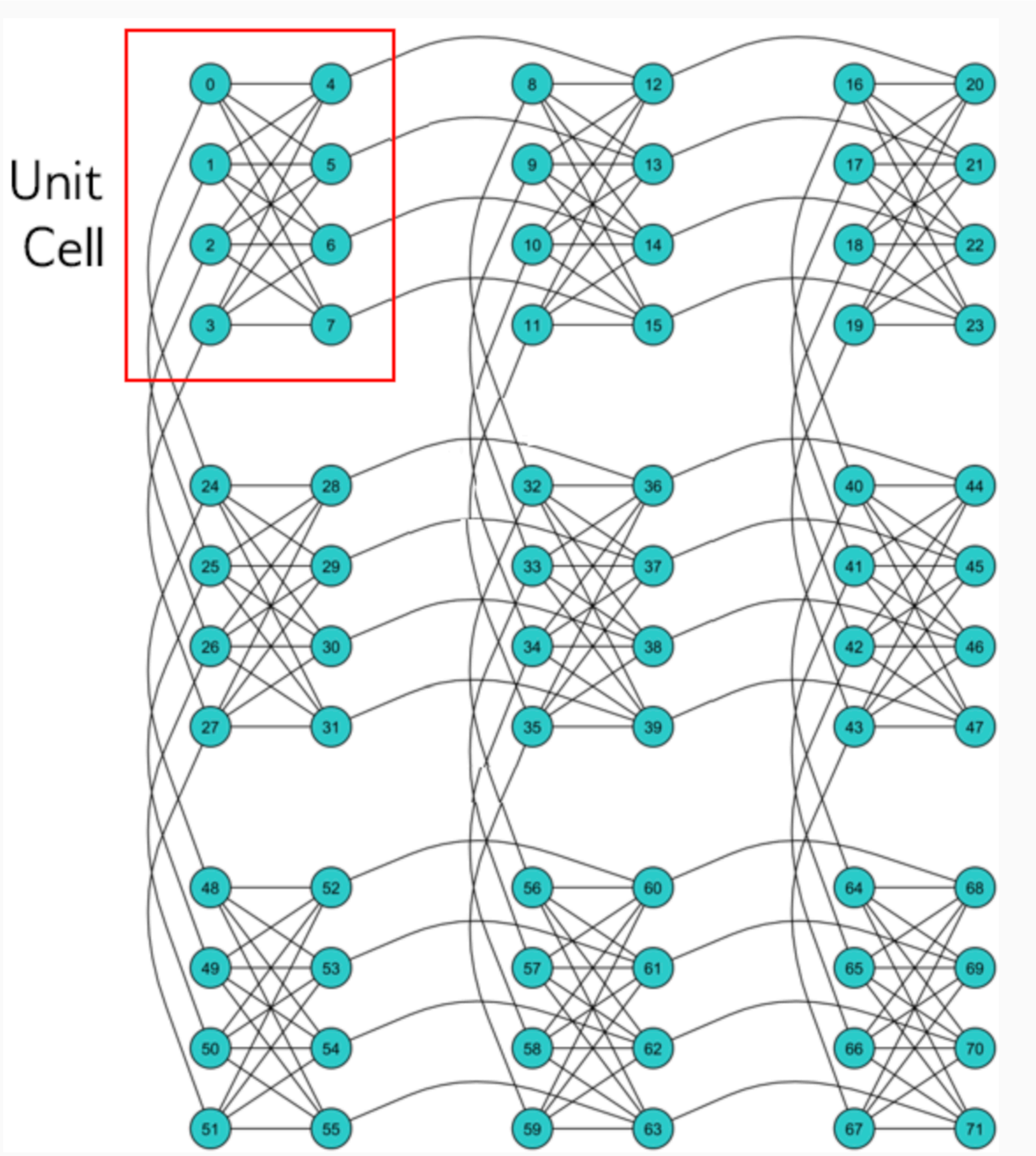


# Chimera Architecture



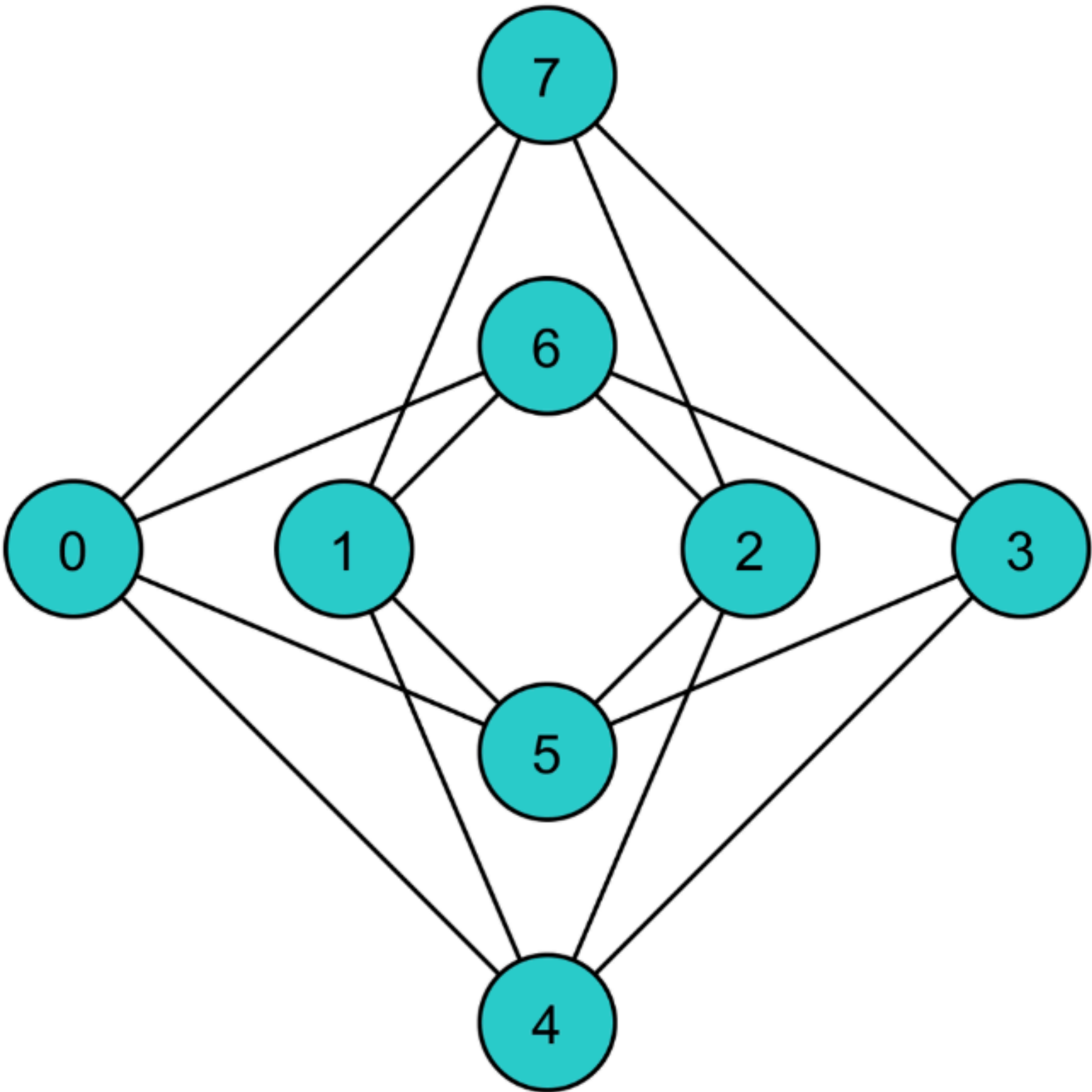
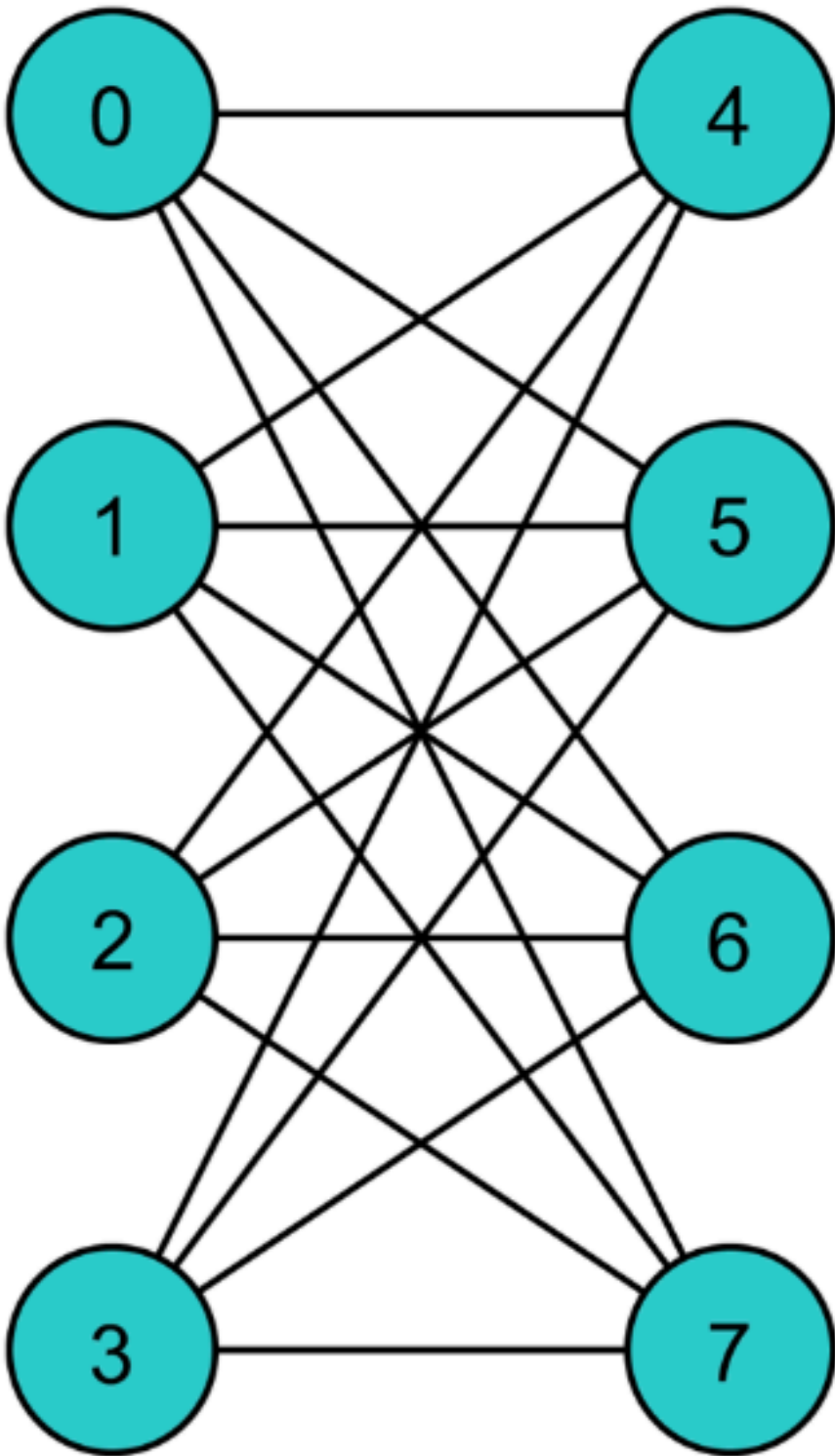
CHIMERA ARCHITECTURE

# Unit Cells



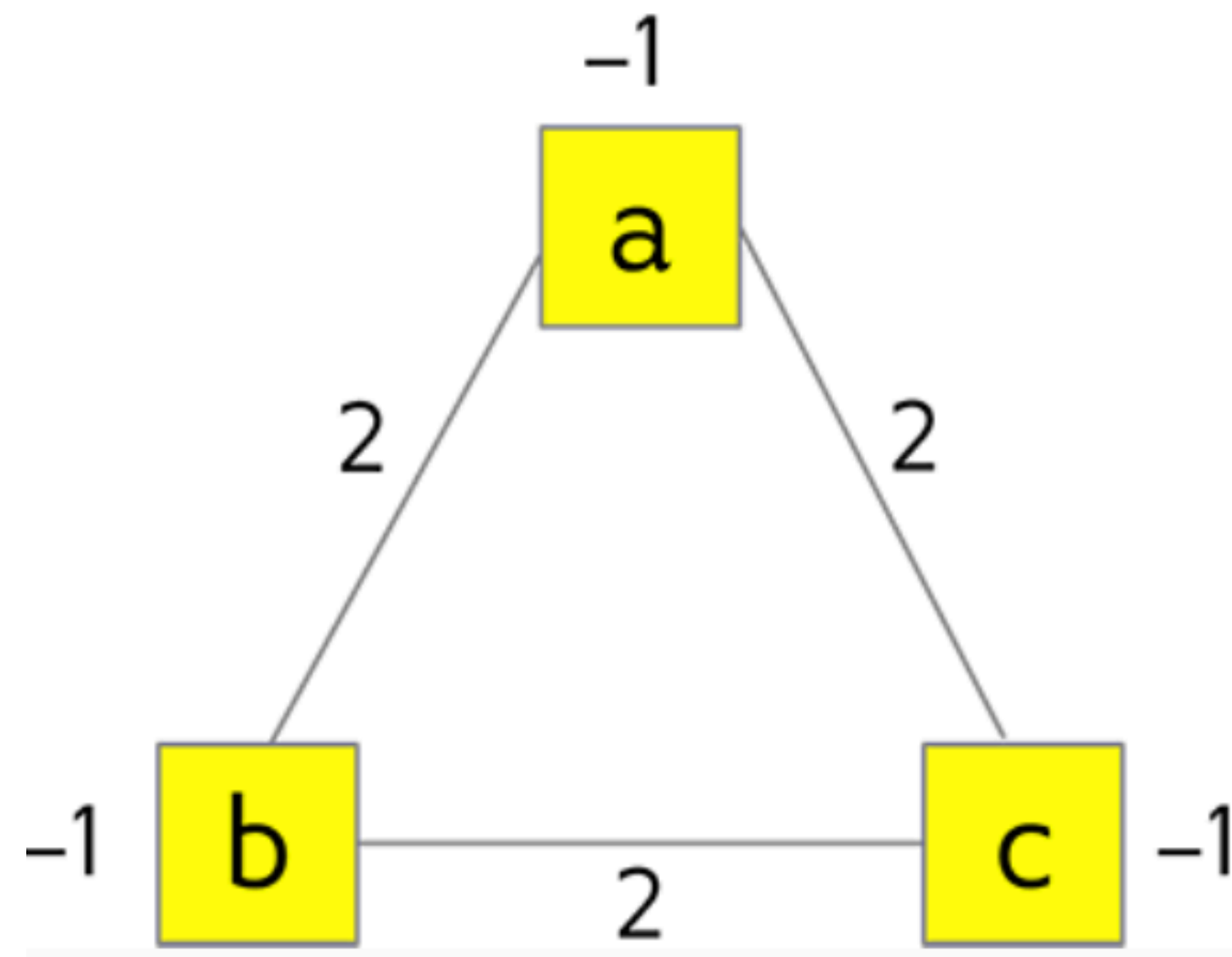


# Unit Cells

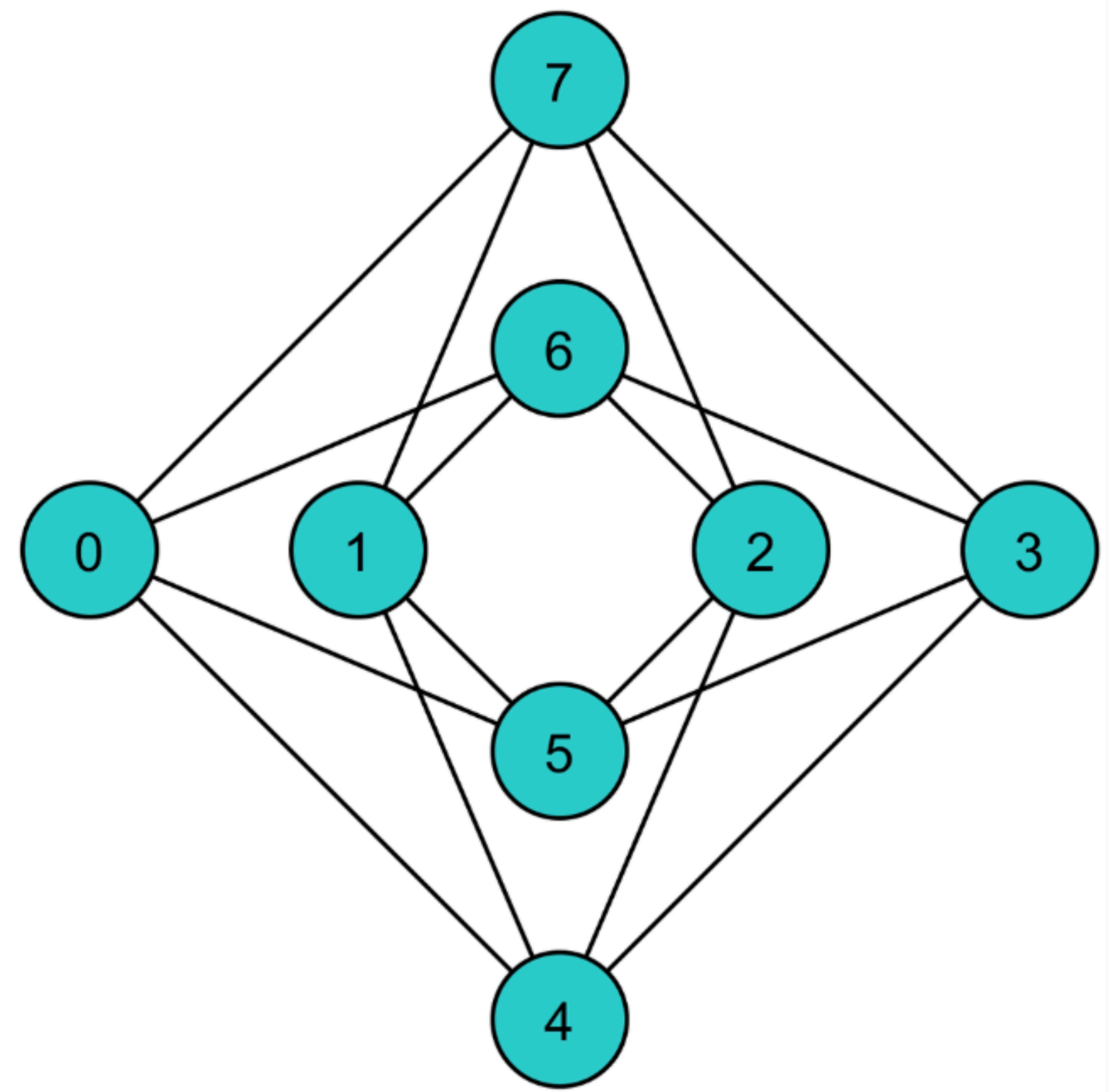




# Minor Embedding

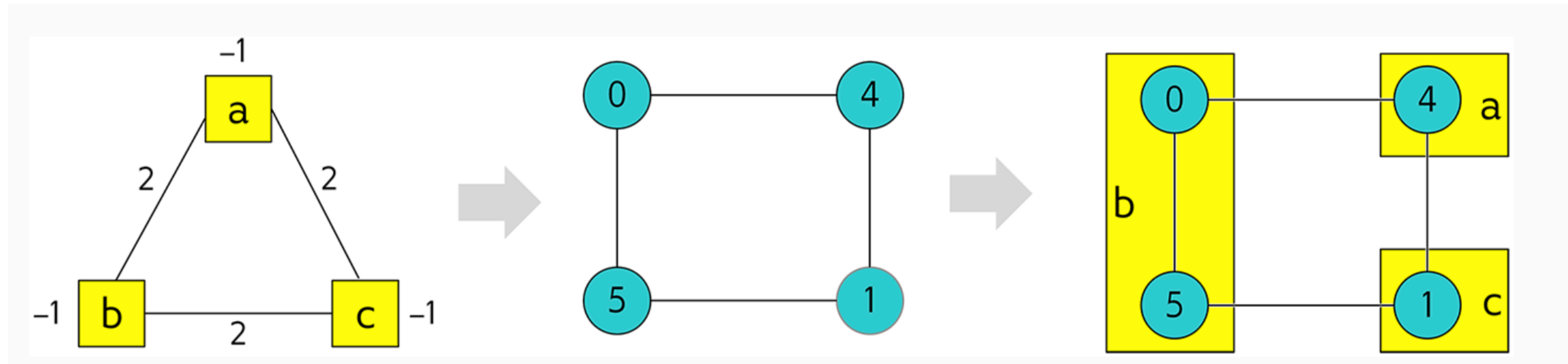


?





# Minor Embedding





## DIVIDING LARGE PROBLEMS

# Divide and conquer

- Problems may be too large for the number of qubits and couplers available on the quantum processor
- This can be solved by **dividing** the problem into smaller sections, solving each section individually and stitching the results back together
- See the DWave Problem Solving Handbook for examples of this
  - [https://docs.dwavesys.com/docs/latest/doc\\_handbook.html](https://docs.dwavesys.com/docs/latest/doc_handbook.html)





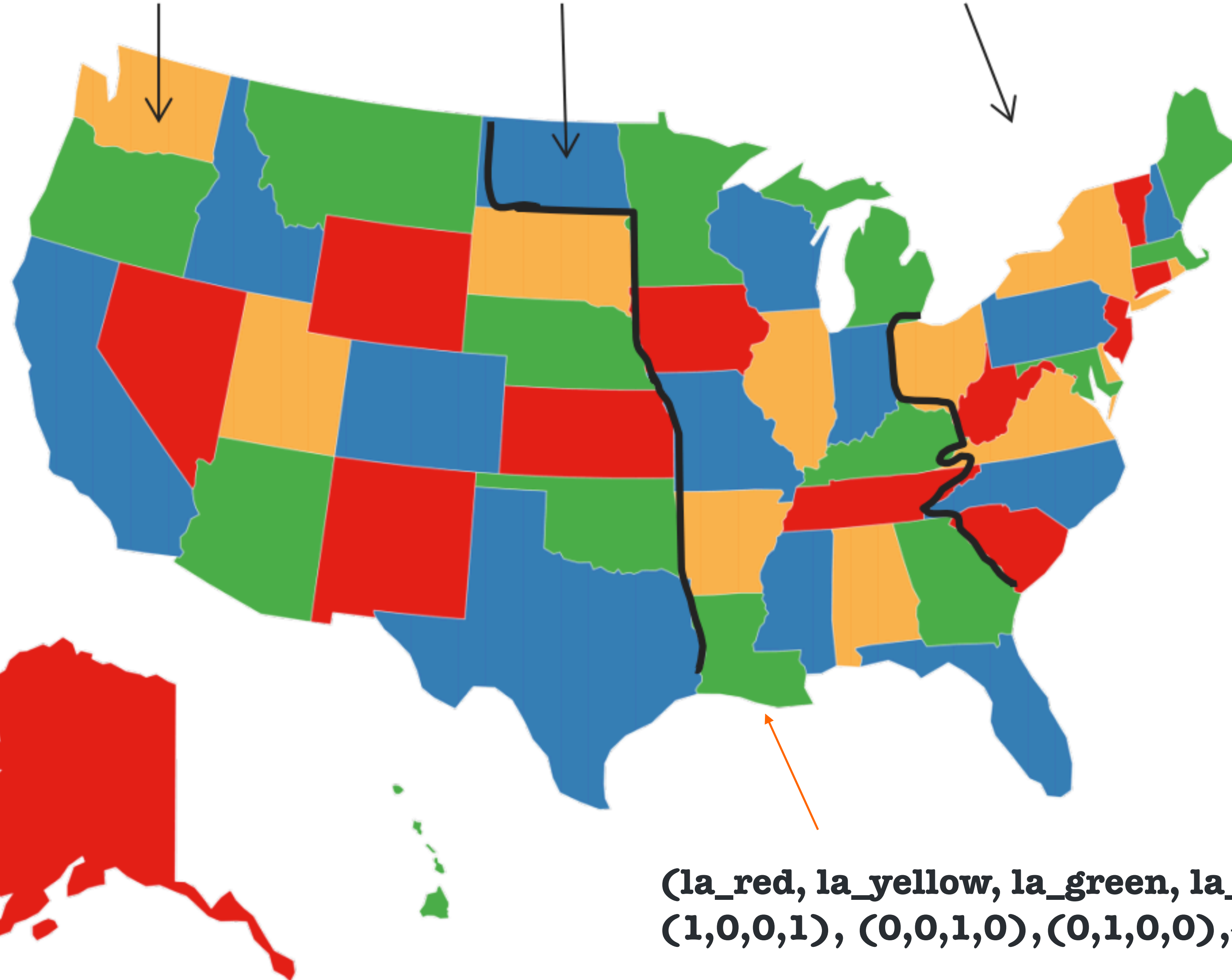




chunk 1

chunk 2

chunk 3

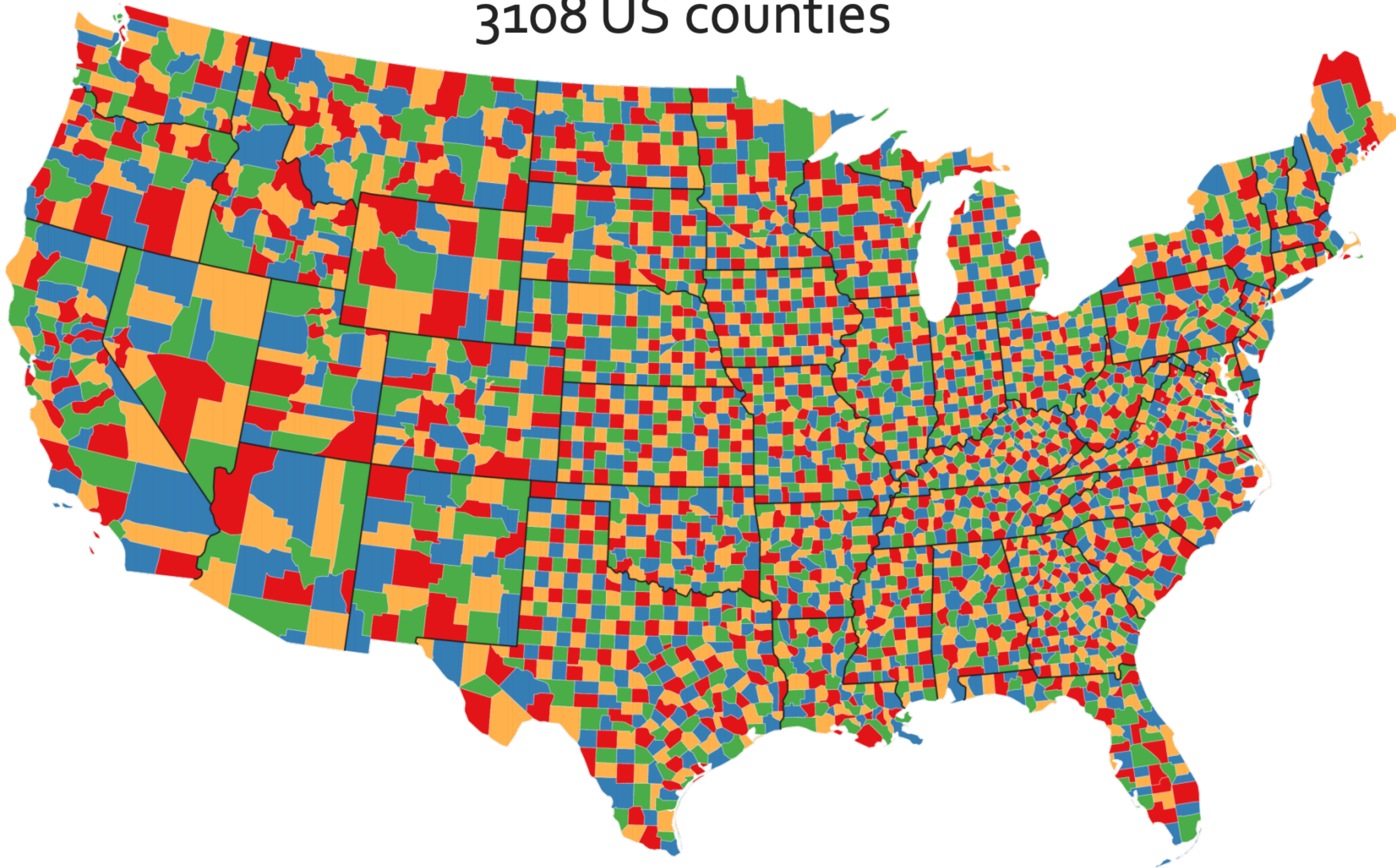


- Calculate chunk 1
- Use colorings as **bias** for calculating chunk 2
- .. repeat

$(la\_red, la\_yellow, la\_green, la\_blue)$  can only be  $(1,0,0,1), (0,0,1,0), (0,1,0,0), (0,0,0,1)$



3108 US counties

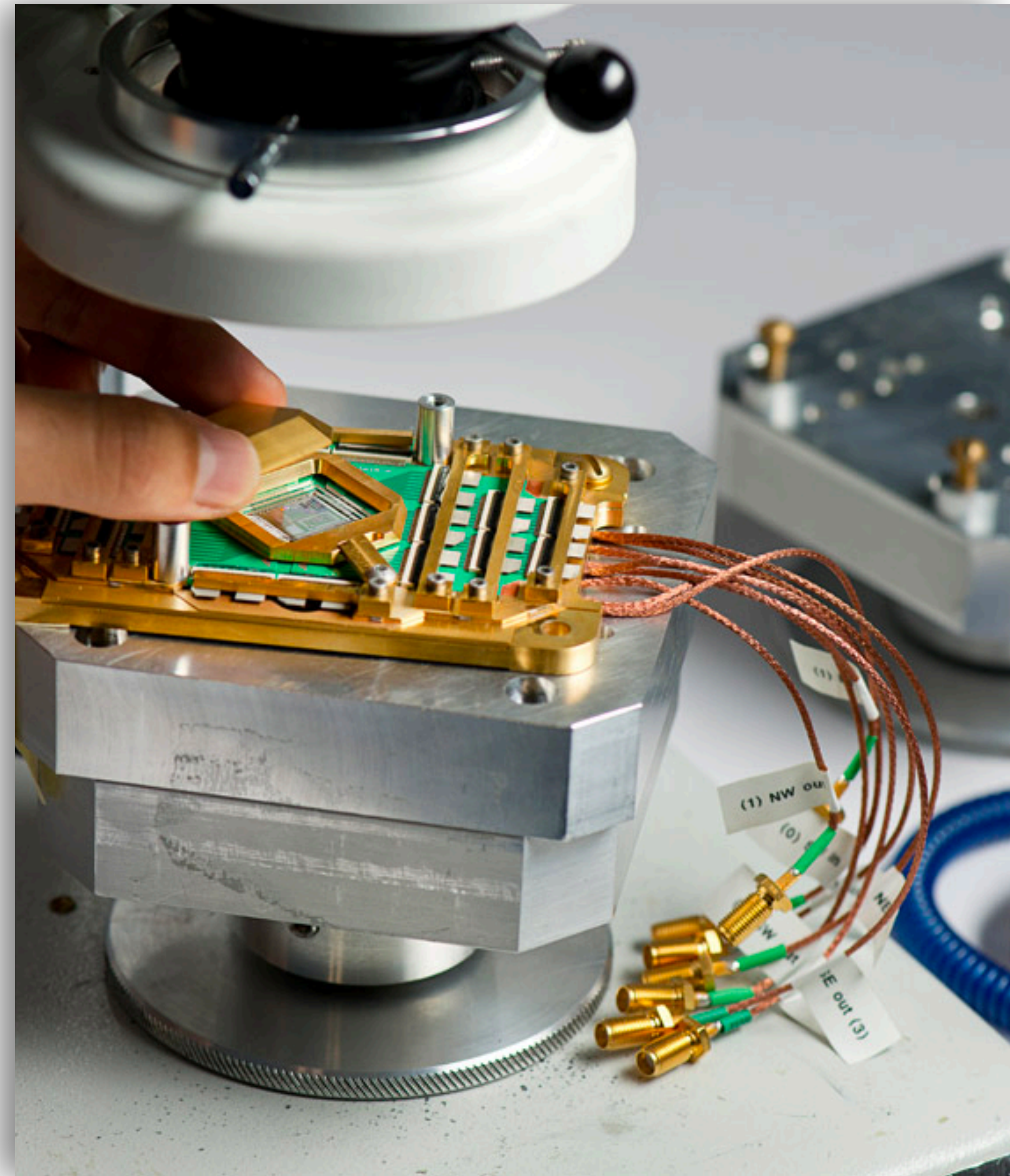




NEXT STEPS

# Resources

- Take the Leap !
  - <https://www.dwavesys.com/take-leap>
- GOTO Masterclasses
  - <https://gotocph.com/2020/pages/offseasonmasterclasses>







# Thank you

NIELS STHEN HANSEN  
[NSH@TRIFORK.COM](mailto:NSH@TRIFORK.COM)